IQSE/AMO SEMINAR

TEXAS A&M UNIVERSITY

Near horizon conformal properties and HBAR universality

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Outline

- Motivation
- Brick-wall model by ‘t Hooft (1985).
- Near-horizon CQM approach by H.C. and C.O. (2005).
- Scully, Fulling et al. (2018).
- Near-horizon approach to acceleration radiation by H.C., A.C., C.O. (2020).
- Acceleration radiation for Kerr metric by Azizi et al. (2020)

Motivation

1975-1976

Hawking discovered black holes emit thermal radiation and the entropy of the radiation is proportional to the area of the black hole.

$$S_{\mathrm{BH}} = \frac{1}{4} A_{\mathrm{BH}}$$

1984

‘t Hooft showed by microstate counting that the thermal atmosphere around a black hole gives a divergent entropy but the entropy is proportional to the area. – Brick-wall approach.

1997

Strominger showed that the conformal symmetry near the horizon for extremal black holes give rise to the area-entropy relation.

2005-2006

Ordonez and Camblong used near-horizon scale symmetry (conformal quantum mechanics) to derive ‘t Hooft’s result.

Scully et al. cavity QED approach to Unruh effect.

2018

Scully et al. developed the quantum optics approach to find the entropy change of a field due to atoms freely falling into a Schwarzschild BH.

Review: ‘t Hooft brick-wall BH entropy model (1985)

- He found a divergent near-horizon contribution to the entropy of the form

$$S = \alpha(a,T)\, A_H$$

$$\alpha(a,T) \sim \frac{1}{a}$$

Area of horizon

$a =$ Small distance cutoff from horizon

If $T = T_H = \dfrac{1}{8\pi M}$, and $a \sim$ Planck length, then α can chosen to be 1/4, i.e.,

$$\boxed{S = \frac{A_H}{4} = S_{BH}\,.}$$

Review: Near-horizon CQM approach, H.C. and C.O. (2005)

- Generalized **D-dimensional** Schwarzschild BH metric

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega^2_{(D-2)}$$

Largest root of $f(r_+) = 0$: Outer event horizon.

Includes **Reissner-Nordström** in D dimensions (and possible extensions with a cosmological constant)

- The scalar field satisfies the K-G equation:

$$S = \frac{1}{2}\int d^D x\sqrt{-g}\,[g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi + \mathrm{m}^2\Phi^2 + \xi R\Phi^2]$$

$$\Rightarrow \quad \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\Phi\right) - \left(\mu^2 + \xi R\right)\Phi = 0\,.$$

- Quantization of the scalar field (asymptotically flat):

$$\Phi(t,r,\Omega) = \sum_s \left[a_s\phi_s(r,\Omega)e^{-i\omega_s t} + \text{ H.c.}\right]\,.$$

Review: Near-horizon CQM approach, H.C. and C.O. (2005)

- Angular decomposition (D dimensions) + Liouville transformation leads to Schrodinger- like Eq. for radial part.

$$\phi_{nlm} = \mathrm{Y}_{\mathrm{lm}}(\Omega)\chi(r)u_{nl}(r)e^{-i\omega_{nl}t}$$

A function r that generates the Liouville transformation: $\chi(r) = f(r)^{-1/2}r^{-(D-2)/2}$.

$$u''(r) - V_{\mathrm{eff}}\left(r, \omega, \mu, \alpha_{l,D}\right) u(r) = 0\,.$$

- In the Near Horizon (NH) approximation: $r = r_+ + x\,.$

$$\boxed{V_{\mathrm{eff}}(r, \omega, \mu, \alpha_{l,D} \overset{(\mathcal{H})}{\sim} -\left(\Theta^2 + \frac{1}{4}\right)\frac{1}{x^2} + \frac{\alpha_{l,D}}{2\kappa r_+^2}\frac{1}{x}}$$

Conformal Quantum Mechanics (CQM) type potential.

$$\Theta = \frac{\omega}{2\kappa}\,, \qquad \kappa = \frac{f'_+}{2} \equiv \text{ surface gravity}$$

Review: Near-horizon CQM approach, H.C. and C.O. (2005)

- CQM contribution leads to same divergence in S found by 't Hooft.
- Temperature is obtained from the conformal parameter Θ: $\boxed{T = T_H = \frac{f_+'}{4\pi} = \left(\frac{4\pi\Theta}{\omega}\right)^{-1}.}$
- At a distant cutoff $\sim$ Planck length: $\boxed{S = S_{BH} = \frac{A_{D-2}}{4}.}$

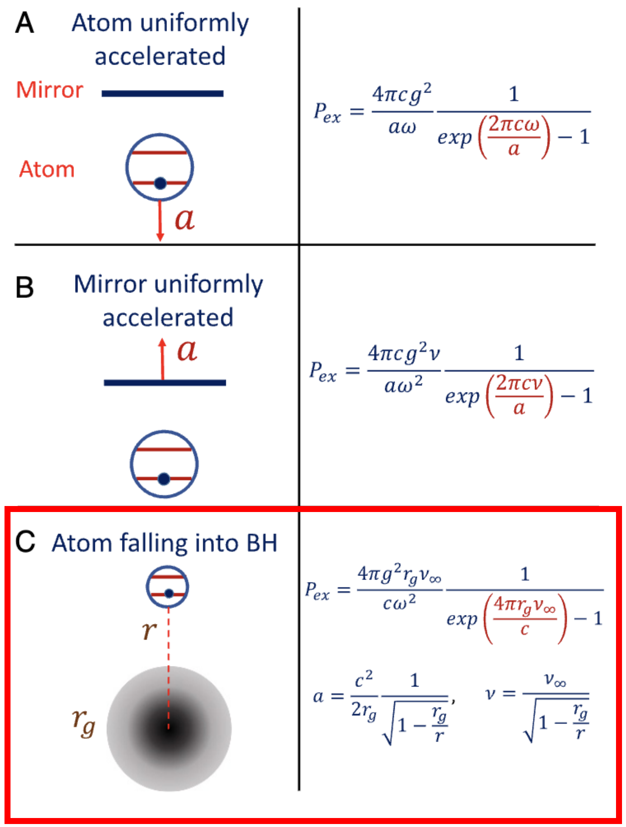
Thermodynamics is dictated by the near-horizon conformal physics!

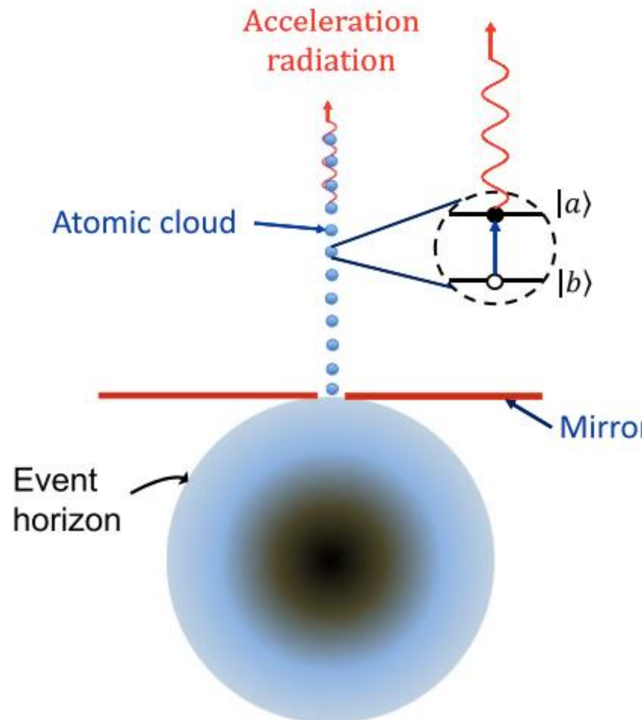
Insights gained:

- Technical: need consider only NH region to find divergence in *S.*
- Allows generalized Schwarzschild D-dimensional metrics.
- Reveals deep connections with conformal symmetry potentials.

Review: Scully, Fulling et al. (2018)

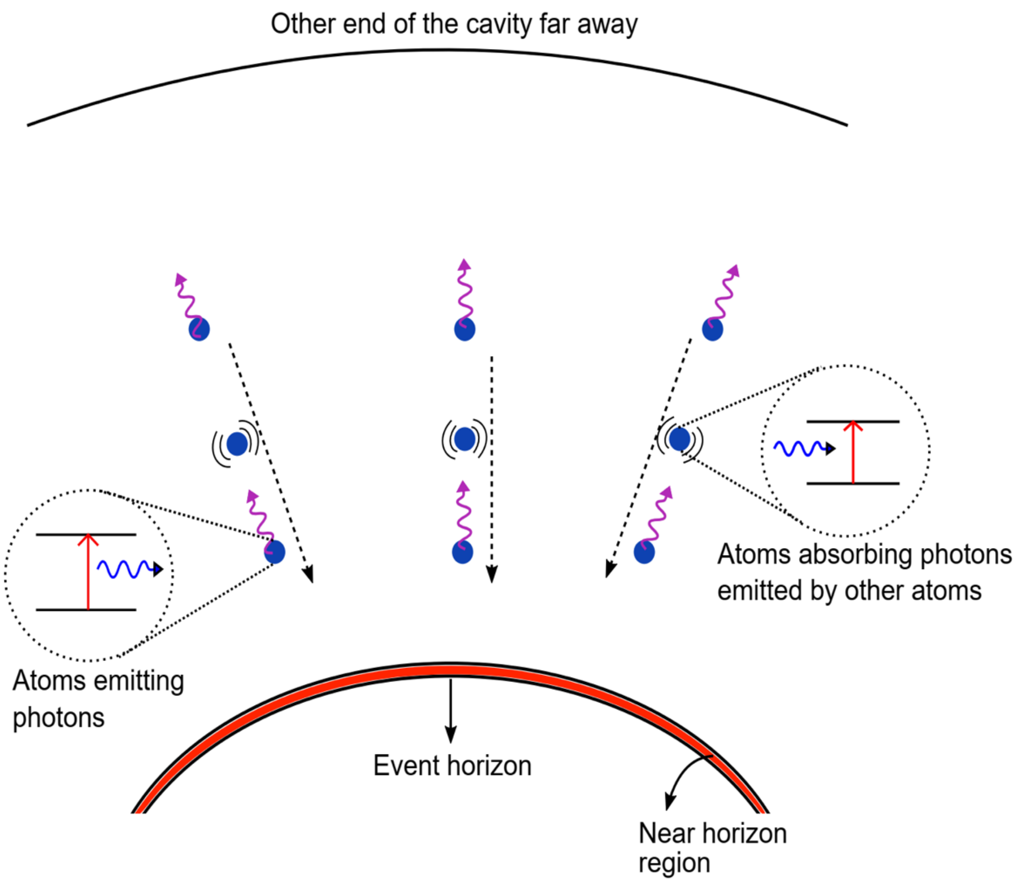
What is the probability of the atomic detector clicking ?





Set-up of the problem

- The presence or absence of particle depends on the state of the observer. In an incompatible vacuum an observer can detect particles.
- Two-state atoms freely falling into black hole are inertial observers which if falling observers which if falling through a Boulware vacuum (an accelerated vacuum) can get excited.
- During this excitation, they also emit a photon (a non-trivial process) which can be absorbed by other atoms.



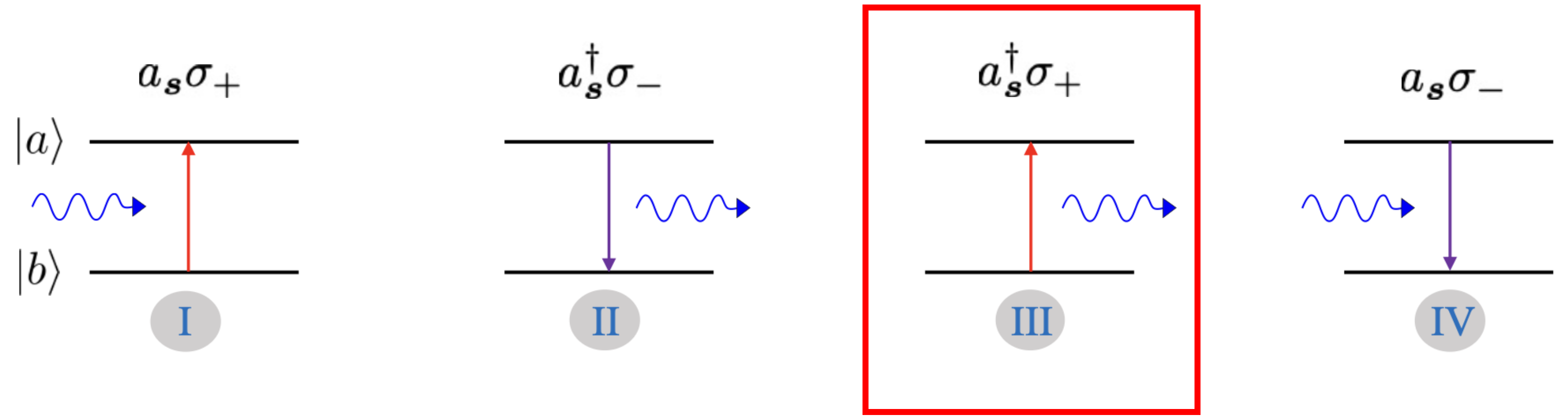
Interaction of fields with atoms: quantum optics

Atoms interact with field via dipole interaction.

$$H = H_0 + V_I$$

$$H_0 = \sum_{i=1}^{2} E_i |i\rangle\langle i| + \sum_k \hbar\omega_k \left(a_k^\dagger a_k\right)$$

$$V_I \equiv H_I = \sum_{s} g_{s} \left[a_{s}\sigma_+ e^{-i(\omega_{s}-\nu)t} + a_{s}^\dagger \, \sigma_- e^{i(\omega_{s}-\nu)t} + a_{s}^\dagger \sigma_+ e^{i(\omega_{s}+\nu)t} + a_{s}\sigma_- e^{-i(\omega_{s}+\nu)t}\right]$$



Probability of excitation of the atom with simultaneous emission of a photon.

Acceleration radiation in the Schwarzschild background

Schwarzschild background:

$$ds^2 = -\left(1 - \frac{r_+}{r}\right) dt^2 + \left(1 - \frac{r_+}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Excitation probability (3rd term):

Both atom and field in excited state

Both atom and field in ground state

$$P_{\rm ex} = \left| \int_{-\infty}^{\infty} dt \, \langle 1_k, a | \hat{V}_I(t) | 0_k, b \rangle \right|^2$$

$$= g^2 \left| \int_{-\infty}^{\infty} d\tau \; e^{i\nu\tau} e^{i\omega t(\tau)} \mathcal{E}_k^*(r(\tau)) \right|^2$$

Initial condition: Starts radial free fall from infinity from rest.

- Solve Klein-Gordon equation: $\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \mathcal{E}_k \right) = 0 \, .$
- Solve geodesic equations.

$$P_{\rm ex} = \frac{4\pi r_s g^2 \omega}{\nu^2} \frac{1}{e^{4\pi r_s \omega} - 1}$$

With the approximation for typical atomic frequencies: $\nu \gg \omega$.

- Atom detects particle in thermal bath with temperature: $T_H = \frac{1}{4\pi r_s}$.

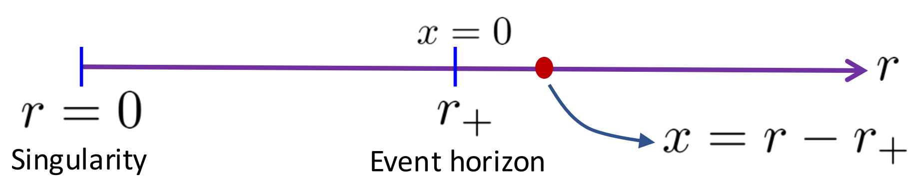
Scully, Fulling et al, PNAS, 2018

Near-horizon approach to acceleration radiation (H.C., A.C., C.O., 2020)

Questions:

- What is the contribution of the event horizon in the acceleration radiation?

  We use a near-horizon (NH) expansion in terms of a NH variable: $x = r - r_+$.



- What about general background metric?

  $$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2_{(D-2)} \ .$$

- What about general initial conditions?

  Free fall starts at distance r_0 with initial energy/mass *e*.

Conformal quantum mechanics

- The scalar field is quantized as:

$$\phi(t,r,\Omega)=\sum_{n,l,m}\left[a_{nlm}\phi_{nlm}(r,\Omega)e^{-i\nu_{nl}t}+a^{\dagger}_{nlm}\phi^{*}_{nlm}(r,\Omega)e^{i\nu_{nl}t}\right]\ .$$

- Separation of variables: $\phi_{nlm}(r,\Omega)=Y_{lm}(\Omega)\chi(r)u_{nl}(r)$

- NH radial equation $x=r-r_+$:

$$u''(x)-V_{eff}(x)u(x)=0$$

$$V_{eff}(x)=-\frac{\lambda_{eff}}{x^2}\ ,\qquad \lambda_{eff}=\frac{1}{4}+\Theta^2\ ,\qquad \Theta=\frac{\omega}{f'_+}$$

- Field modes:

$$\phi(r,t)\overset{(\mathcal{H})}{\sim}\frac{1}{\sqrt{f'_+}}r_+^{-(D-2)/2}x^{i\Theta}e^{-i\omega t}$$

$$\overset{(\mathcal{H})}{\sim}x^{i\Theta}e^{-i\omega t}=e^{-i\omega(t-\ln x/f'_+)}.$$

Camblong, Chakraborty, Ordonez, PRD, 2020

Conformal quantum mechanics

- NH geodesic equations:

$$\tau = -\int \frac{dr}{\sqrt{e^2 - f(r)\,(1 + l^2/r^2)}} \overset{(\mathcal{H})}{\sim} -\frac{x}{e} + \text{ const. } + \mathcal{O}(x^2)$$

$$t = -\int dr \, \frac{e/f(r)}{\sqrt{e^2 - f(r)\,(1 + l^2/r^2)}} \overset{(\mathcal{H})}{\sim} -\frac{1}{f'_+} \ln x - Cx + \text{ const. } + \mathcal{O}(x^2)$$

C is a constant dependent on the conserved quantities given by $C = \frac{1}{2}\left[\frac{1}{e^2}\left(1 + \frac{\ell^2}{r_+^2}\right) - \frac{f''_+}{(f'_+)^2}\right]$.

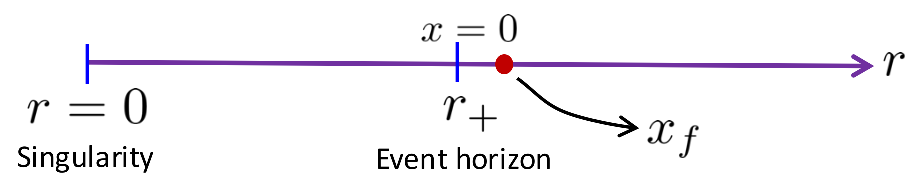
- Simplifies our calculations significantly.

$$P_{\text{exc}} = g^2 \left| \int d\tau \, \phi^*(r(\tau), t(\tau)) e^{i\nu\tau} \right|^2 = \frac{g^2}{e^2} \left| \int_0^{x_f} dx \, x^{-i\Theta} e^{i\omega(-\ln x/f'_+ - Cx)} e^{-i\nu x/e} \right|^2$$

$$= \frac{g^2}{e^2} \left| \int_0^{x_f} dx \, x^{-i\sigma} e^{-isx} \right|^2$$

$$\sigma = 2\Theta = \frac{2\omega}{f'_+} = \frac{\omega}{\kappa}, \qquad s = C\omega + \frac{\nu}{e} = \frac{\omega}{2}\left[\frac{1}{e^2}\left(1 + \frac{\ell^2}{r_+^2}\right) - \frac{f''_+}{(f'_+)^2}\right] + \frac{\nu}{e}, \qquad \kappa = f'_+/2.$$

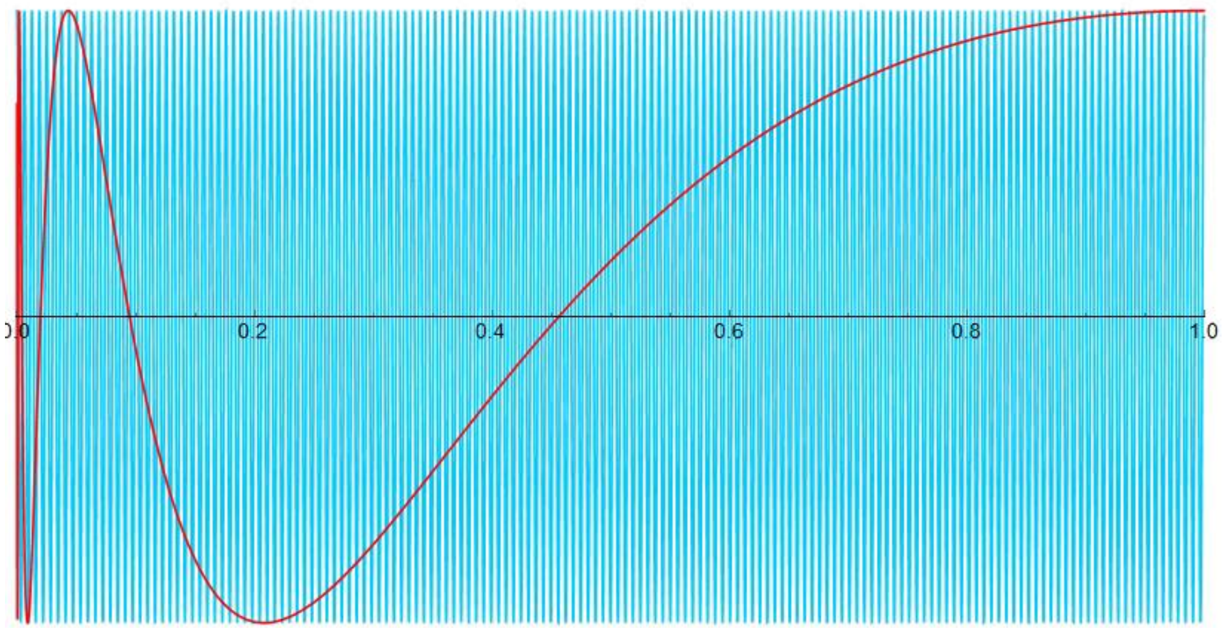
x_f: Upper limit of the integration that signifies the boundary of a region where the near-horizon approximation is valid.



Near-horizon conformal aspects

$$P_{\rm ex} = \frac{g^2}{e^2}\left|\int_0^{x_f} dx\, \underbrace{x^{-i\omega/\kappa}}_{f_1(x)}\, \underbrace{e^{-isx}}_{f_2(x)}\right|^2 , \qquad s = C\omega + \frac{\nu}{e}\,.$$

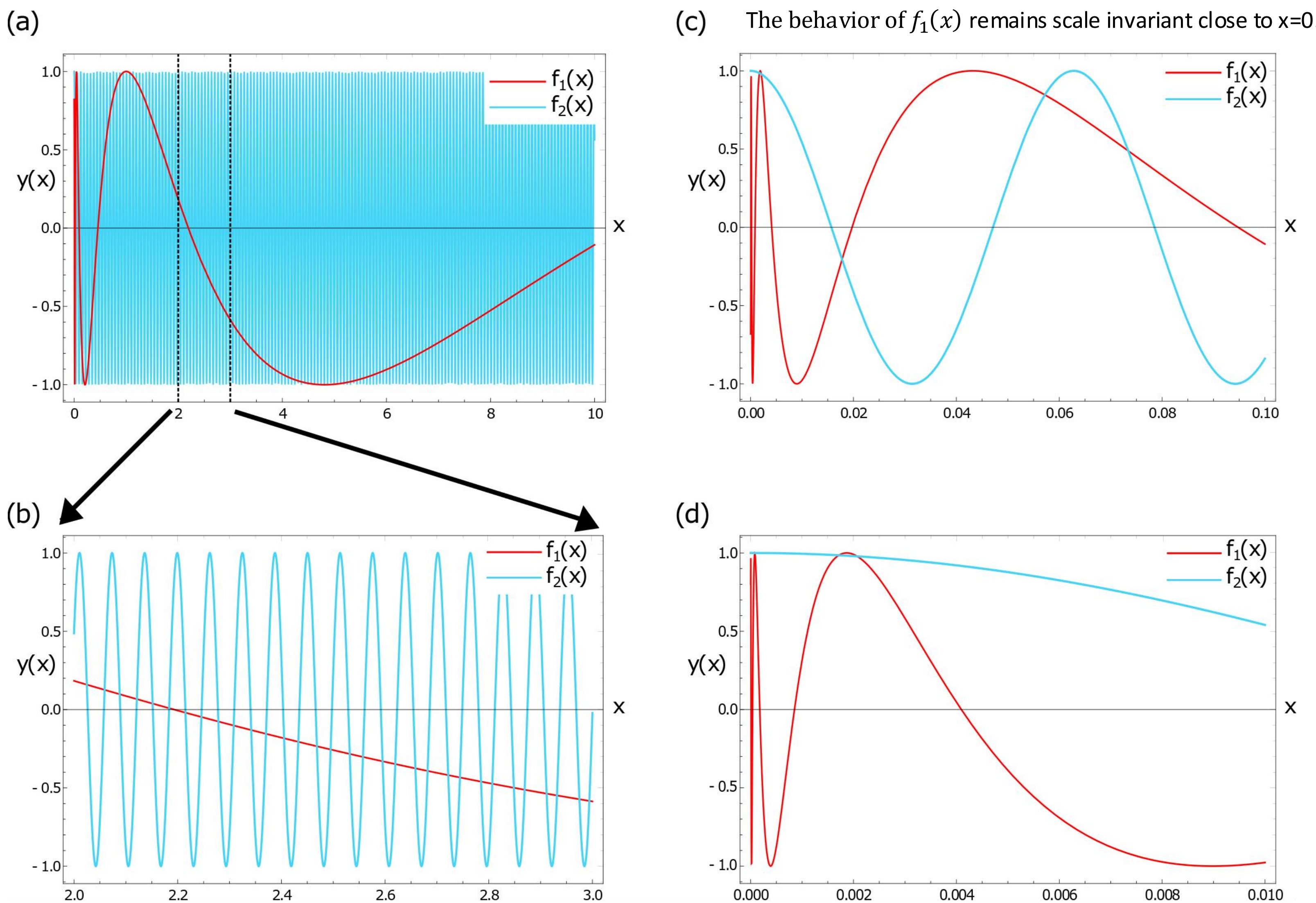
All aspects of the NH physics rely on $f_1(x)$.



Camblong, Chakraborty, Ordonez, PRD, 2020

Near-horizon conformal aspects

(c) The behavior of $f_1(x)$ remains scale invariant close to x=0.



Slow variation of $f_1(x)$ compared to $f_2(x)$

Camblong, Chakraborty, Ordonez, PRD, 2020

Near-horizon conformal aspects

- Allows us to change the upper limit of the integral.

$$P_{\rm ex} = \frac{g^2}{e^2} \left| \int_0^\infty dx \, x^{-i\omega/\kappa} \, e^{-isx} \right|^2 , \qquad s = C\omega + \frac{\nu}{e} .$$

- In the limit $\nu >> \omega$

$$\boxed{P_{\rm ex} = \frac{2\pi g^2 \omega}{\kappa \nu^2} \frac{1}{e^{2\pi\omega/\kappa} - 1} .}$$

where $\kappa = \dfrac{f'(r_+)}{2}$

(surface gravity)

- Thermal spectrum with temperature: $T_H = \frac{\kappa}{2\pi}$.

Camblong, Chakraborty, Ordonez, PRD, 2020

Answer to previous questions.

- What is the contribution of the event horizon in the acceleration radiation?
  - ✓ The thermal nature of the radiation is completely governed by the **physics near the event horizon**.
  - ✓ Conformal nature of the waves near the horizon is extremely important in creating the **thermal acceleration radiation**.
- What about general background metric?
  - ✓ We can generalize our results to any D-dimensional generalized Schwarzschild black hole including Reissner-Nordstorm metric, as well as combinations of these with a cosmological constant, and blackhole solutions with additional charges.
- What about general initial conditions?
  - ✓ Our results are **independent of the initial conditions**.

Q: What about a stationary metric?

Rotating black hole: Kerr metric.

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a\sin^2\theta d\phi)^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)d\phi - adt\right]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

$$\Delta = r^2 - 2Mr + a^2$$
$$\rho^2 = r^2 + a^2\cos^2\theta.$$

With the NH expansion the calculation becomes analytically tractable:

$$P_{\rm ex} = g^2 \left| \int d\tau \; e^{i\tilde{\omega}(t-r_*)} e^{i\nu\tau} \right|^2 ,$$

where $\tilde{\omega} = \omega - m\Omega_H$.
r_* is the tortoise coordinate.

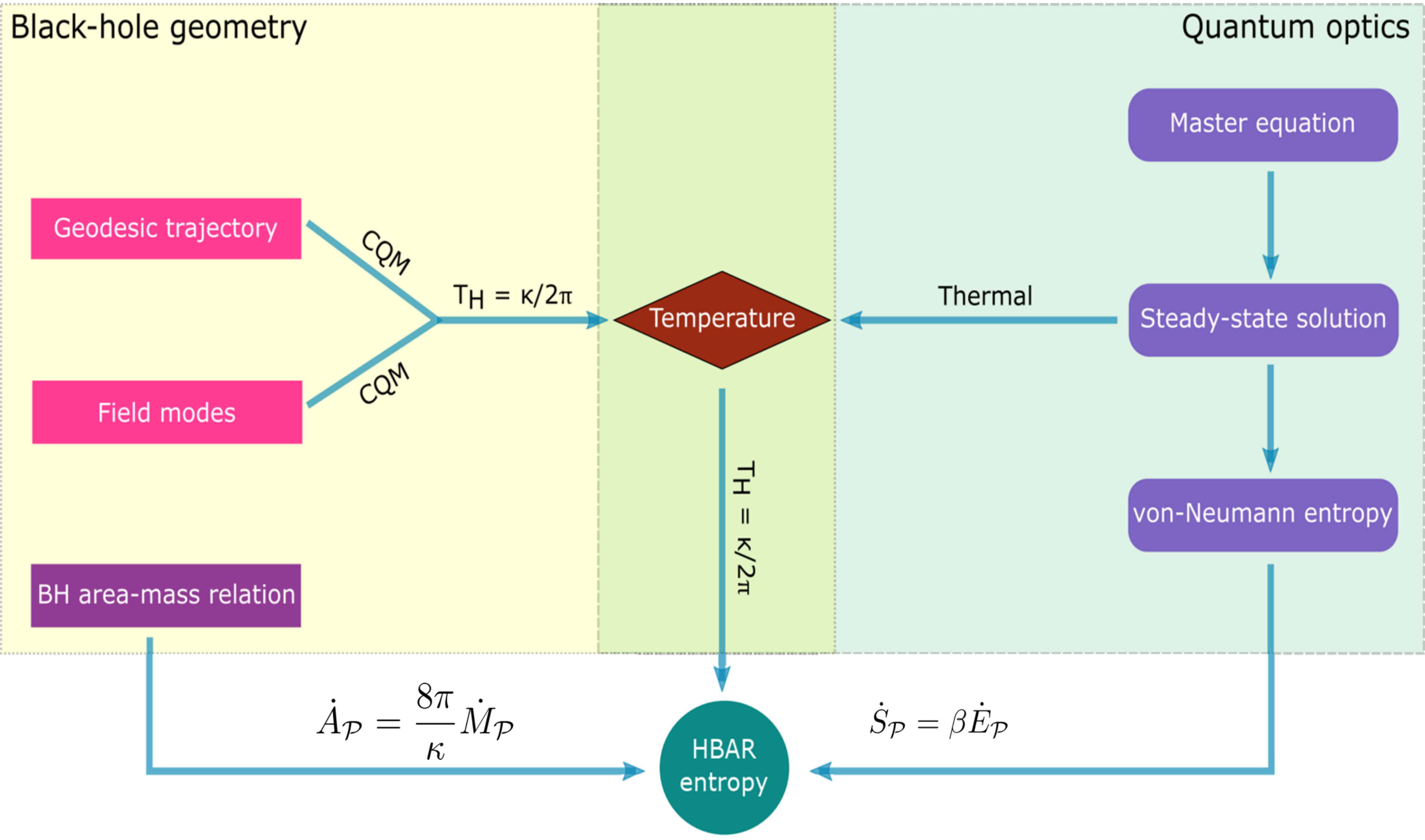
Furthermore, $t - r_* = -\frac{1}{\kappa}\ln x - Cx$, and $\tau = -\frac{r_+^2}{c_0}x$, where $\kappa = \frac{r_+ - r_-}{2(r_+^2+a^2)}$.

$$\boxed{P_{\rm ex} = \frac{2\pi g^2\tilde{\omega}}{\kappa\nu^2}\frac{1}{e^{2\pi\tilde{\omega}/\kappa} - 1}}$$

- The acceleration radiation is thermal in nature with temperature $T_H = \frac{\kappa}{2\pi}$.

Azizi et al, PRD 2021.

Quantum optics and HBAR entropy: Flow chart



Summary and future work

- The main contribution to this excitation probability is provided by the near-horizon physics of the black hole.

- The conformal nature of the integrand plays a major role in determining the thermal nature of the spectrum.

- The near-horizon expansion allows us to extend the formalism to the most general black hole metrics and the most general initial conditions which otherwise would have been almost impossible to calculate analytically.

- **We're currently further developing the universality and robustness of HBAR entropy by considering BTZ black holes.**

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